

Kane Symposium
Warm-up Act
for Headliners to come
January 19, 2007

Examples of the Zeroth Theorem of the History of Science

John David Jackson

Preamble

The Victor Weisskopf Collegiate Professor of Physics and I
go 'way back - over 1/2 my life and almost 2/3 of Gordy's.
More than most here, I imagine.

BA, Philosophy, U. Minn., 1958; a little time at MIT in
Physics; Francis Low sent him to Urbana; MS, U. Ill. 1961;
Ph.D., 1963.

Played an important role in "Classical Electrodynamics."

He insisted that I make a very thorough index !



24 year old grad student Gordon L. Kane, attending Gina and David Hafemeister's wedding, summer 1961

To show that Gordy did not initially appear in full-blown supersymmetric mode, I give you the first pages of his Ph.D. thesis.

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ANALYSIS OF THE ANGULAR DISTRIBUTION OF
PION-NUCLEON SCATTERING WITHIN THE
FRAMEWORK OF THE MANDELSTAM
REPRESENTATION

THESES

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Physics
in the Graduate College of the
University of Illinois, 1963

Urbana, Illinois

UNIVERSITY OF ILLINOIS
THE GRADUATE COLLEGE

July 15, 1963

PREPARED UNDER MY

Ion-Nucleon Scattering

... the Framework of the Mandelstam Representation

BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF Doctor of Philosophy in Physics

J. D. Jackson
in Charge of Thesis

G. M. Kinsey
Head of Department

Recommendations concurred in by

<i>Maurice Riesz</i>	Committee on Final Examination
<i>Arthur J. Lee</i>	
<i>H. W. Wyld, Jr.</i>	
<i>John D. Jackson</i>	

I Recommend for doctor's degree but not for master's.

1963

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Zeroth Theorem of the History of Science

What is it ?

In a July 24, 2006 column entitled *Fremde Federn, Im Gegenteil* in *Die Welt* (Berlin), E. P. Fischer states

"Das Nullte Theorem der Wissenschaftsgeschichte lautet, dass eine Entdeckung (Regel, Gesetzmässigkeit, Einsicht), die nach einer Person benannt ist, nicht von dieser Person herrührt."

A schoolboy's translation is

"The zeroth theorem of the history of science reads that a discovery (rule, regularity, insight) named after someone, (often?) did not originate with that person."

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Fischer, a German historian of science, cites some examples:

Avogadro's number (6.022×10^{23}) was first determined by Loschmidt in 1865 (although Avogadro had found in 1811 that any gas at NTP had the same number of molecules per unit volume)

Halley's comet was known 100 years before Halley's birth (but Halley noted its appearance at regular intervals and predicted correctly its next appearance)

Olber's paradox (1826) was discussed by Kepler (1610), and by Halley and Cheseaux in the 18th century.

And others

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Five Examples of the Zeroth Theorem in Physics

Some may be familiar, some may not.

Example No. 1

**Lorentz Condition, $\partial_\mu A^\mu = 0$,
for the electromagnetic potentials**

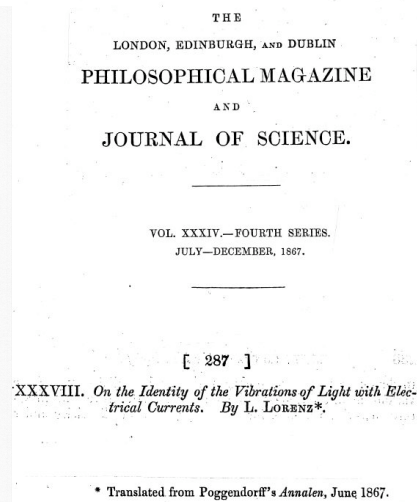
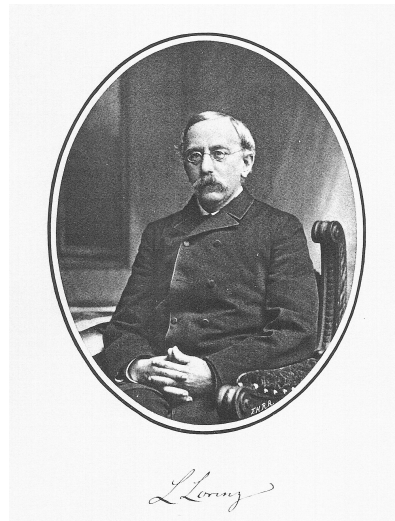
Hendrik Antoon Lorentz, 1904

$$\operatorname{div} \mathbf{a} = -\frac{1}{c} \dot{\varphi},$$

Ludvig Valentin Lorenz, 1867

$$\frac{d\bar{\Omega}}{dt} = -2 \left(\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz} \right).$$

Let's look at a little of Ludvig Valentin Lorenz's paper :



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The equations (A) with $k = 1$ are the equations for the electric field in terms of the potentials. Below, the usual expressions for the components of the vector potential in terms of the retarded sources.

LVL understood the equivalence of different potentials

Retardation is important! But Maxwell (1868) objected to Lorenz's retarded potentials !

$$\left. \begin{aligned} u &= -2k \left(\frac{d\Omega}{dx} + \frac{4}{c^2} \frac{da}{dt} \right), \\ v &= -2k \left(\frac{d\Omega}{dy} + \frac{4}{c^2} \frac{d\beta}{dt} \right), \\ w &= -2k \left(\frac{d\Omega}{dz} + \frac{4}{c^2} \frac{d\gamma}{dt} \right), \end{aligned} \right\} \quad (A)$$

where, for brevity's sake, we put

$$\begin{aligned} a &= \iiint \frac{dx' dy' dz'}{r} u' \left(t - \frac{r}{a} \right), \\ \beta &= \iiint \frac{dx' dy' dz'}{r} v' \left(t - \frac{r}{a} \right), \\ \gamma &= \iiint \frac{dx' dy' dz'}{r} w' \left(t - \frac{r}{a} \right). \end{aligned}$$

These equations are distinguished from equations (1) by containing, instead of U, V, W , the somewhat less complicated members a, β, γ ; and they express further that the entire action between the free electricity and the electrical currents *requires time to propagate itself*—an assumption not strange in science, and which may in itself be assumed to have a certain degree of probability. For in accordance with the formulæ found, the action in the point $x y z$ at the moment t does not depend on the *simultaneous* condition in the point $x' y' z'$, but on the condition in which it was at the moment $t - \frac{r}{a}$; that is, so much time in advance as is required to traverse the distance r with the constant velocity a .

10

After showing that his retarded potentials yield the known quasi-static results, Lorenz proceeds toward differential equations for the fields. Along the way he finds a relation between the potentials:

The Loren(t)z condition

we obtain

$$\frac{d\bar{\Omega}}{dt} = -2 \left(\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz} \right).$$

which leads to the wave equation for the vector potential

Moreover from (5),

$$\frac{1}{a^2} \frac{d^2 \alpha}{dt^2} = \Delta \alpha + 4\pi u,$$

and in like manner for β, γ . If now these values be substituted in the equations (A), after they have been differentiated in reference to t , and if $c = a\sqrt{2}$, we get

He then arrives at the Ampère-Maxwell equation in component form.

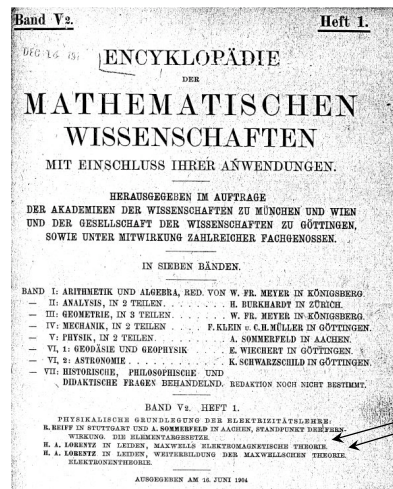
$$\left. \begin{aligned} \frac{1}{4k} \frac{du}{dt} + 4\pi u &= \frac{d}{dx} \left(\frac{d\gamma}{dx} - \frac{d\alpha}{dz} \right) - \frac{d}{dy} \left(\frac{d\alpha}{dy} - \frac{d\beta}{dx} \right), \\ \frac{1}{4k} \frac{dv}{dt} + 4\pi v &= \frac{d}{dx} \left(\frac{d\alpha}{dy} - \frac{d\beta}{dx} \right) - \frac{d}{dz} \left(\frac{d\beta}{dz} - \frac{d\gamma}{dy} \right), \\ \frac{1}{4k} \frac{dw}{dt} + 4\pi w &= \frac{d}{dy} \left(\frac{d\beta}{dz} - \frac{d\gamma}{dy} \right) - \frac{d}{dx} \left(\frac{d\gamma}{dx} - \frac{d\alpha}{dz} \right). \end{aligned} \right\} \quad (8)$$

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Thirty seven years later, in 1904, H. A. Lorentz wrote two encyclopedia articles



Scanned at the American Institute of Physics



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On page 157 of the second article, Lorentz introduces the potentials into the Maxwell equations and arrives at wave equations for the potentials, subject to the

3. Auf die geladene Materie wirkende Kraft. 4. Potentiale. 157

schen skalaren Potentials φ und eines magnetischen Vektorpotentials \mathbf{a} darstellen¹⁸⁾. Es genügen diese Hilfsgrößen den Differentialgleichungen

$$(VII) \quad \Delta \varphi - \frac{1}{c^2} \ddot{\varphi} = -\varrho,$$

$$(VIII) \quad \Delta \mathbf{a} - \frac{1}{c^2} \ddot{\mathbf{a}} = -\frac{1}{c} \varrho \mathbf{v},$$

und es ist

$$(IX) \quad \mathbf{b} = -\frac{1}{c} \dot{\mathbf{a}} - \text{grad } \varphi,$$

$$(X) \quad \mathbf{h} = \text{rot } \mathbf{a}.$$

Zwischen den beiden Potentialen besteht die Relation

$$(2) \quad \text{div } \mathbf{a} = -\frac{1}{c} \dot{\varphi}.$$

Loren(t)z condition (2).

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Example No. 2

Dirac delta function

Paul Adrien Maurice Dirac, 1930

$$\delta(x) = 0 \text{ if } x \neq 0, \quad \int \delta(x) dx = 1$$

Oliver Heaviside, 1895

$$p\mathbf{1}, \text{ where } p = d/dt \text{ and } \mathbf{1} = \delta(t)$$

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But first,

Predecessors of Heaviside

The 19th century saw many utilize limiting forms to produce an impulse in a multiple integral in order to affect a formal calculation:

Cauchy, Poisson (1815) , Hermite (~ 1850-60?)

$${}^{\prime\prime}\delta(t) = \lim_{\lambda \rightarrow \infty} \frac{\lambda}{\pi(\lambda^2 t^2 + 1)}$$

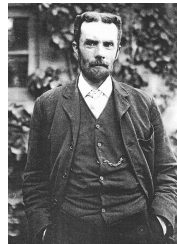
Kirchhoff, Kelvin, Helmholtz (1880's?)

$${}^{\prime\prime}\delta(t) = \lim_{\lambda \rightarrow \infty} \frac{\lambda}{\sqrt{\pi}} \exp(-\lambda^2 t^2)$$

Limits always taken after integration, just as for the usual kluge to get convergence in a touchy integral.

15

From 1894 to 1898, Oliver Heaviside was publishing his operational calculus in *The Electrician*. In the issue of March 15, 1895 he devoted a section to “*Theory of an Impulsive Current produced by a Continued Impressed Force*,” In it was the following partial paragraph:

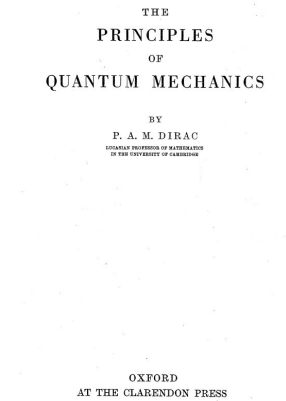


“We have to note that if Q is any function of time, then pQ is its rate of increase. If, then, as in the present case, Q is zero before and constant after $t = 0$, pQ is then zero except when $t = 0$. It is then infinite. But its total amount is Q . That is to say $p1$ means a function of t which is wholly concentrated at the moment $t = 0$, of total amount 1. It is an impulsive function, so to speak. The idea of an impulse is well known in mechanics, and it is essentially the same here. Unlike the function $(p)^{1/2} 1$, the function $p1$ does not involve appeal either to experiment or to generalised differentiation, but only involves the ordinary ideas of differentiation and integration pushed to their limit.”

That's a pretty clear description of a Dirac delta function!

16

Paul Adrien Maurice Dirac
published the first edition
of his famous text in 1930



Trained as an electrical engineer,
(B.Eng., Bristol, 1921), Dirac
would surely have been exposed
to Heaviside's operational
calculus.

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His discussion of the delta function varies somewhat from
edition to edition. Here are excerpts from the 3rd (1947).

15. The δ function

Our work in § 10 led us to consider quantities involving a certain kind of infinity. To get a precise notation for dealing with these infinities, we introduce a quantity $\delta(x)$ depending on a parameter x satisfying the conditions

$$\left. \begin{aligned} \int_{-\infty}^{\infty} \delta(x) dx &= 1 \\ \delta(x) &= 0 \text{ for } x \neq 0. \end{aligned} \right\} (2)$$

To get a picture of $\delta(x)$, take a function of the real variable x which vanishes everywhere except inside a small domain, of length ϵ say, surrounding the origin $x = 0$, and which is so large inside this domain that its integral over this domain is unity. The exact shape of the function inside this domain does not matter, provided there are no unnecessarily wild variations (for example provided the function is always of order ϵ^{-1}). Then in the limit $\epsilon \rightarrow 0$ this function will go over into $\delta(x)$.

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A little later, Dirac gives an alternative definition:

.....

An alternative way of defining the δ function is as the differential coefficient $\epsilon'(x)$ of the function $\epsilon(x)$ given by

$$\left. \begin{aligned} \epsilon(x) &= 0 & (x < 0) \\ &= 1 & (x > 0). \end{aligned} \right\} (5)$$

This definition is explicitly Heaviside's starting point:

$$\square(x) = \mathbf{1}, \quad \square\square(x) = p\mathbf{1}$$

And the descriptions in words of both are strikingly similar (what else could they be?)

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Example No. 3

Schumann resonances of the Earth-ionosphere cavity

Winfried Otto Schumann, 1952

Nicola Tesla, (1900) 1905

George Francis FitzGerald, 1893

20

Schumann resonances ?

The Earth conducts electricity and so does the ionosphere, a plasma-like set of layers at altitudes above 60-100 km. The space between can be idealized as that between a perfectly conducting sphere of radius R and a slightly larger perfectly conducting shell. Electromagnetic waves exist in this space.

Lowest mode ?

$$\omega_0 = O(c/2\pi R) = 7.45 \text{ Hz}$$

Correct answer ?

$$\omega(n) = \sqrt{n(n+1)} \omega_0; n = 1, 2, 3, \dots$$

In real life, poor conductivity and ill-defined boundaries yield resonances at $\omega(n) \approx 5.8 \sqrt{n(n+1)} \text{ Hz}$.



Winfried Otto Schumann

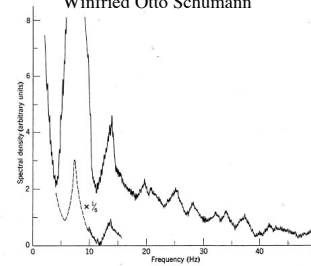


Figure 8.9 Typical noise power spectrum at low frequencies (integrated over 30 s), observed at Larvagsdalen, Norway on June 19, 1965.

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No. 787,412.

Patented April 18, 1905.

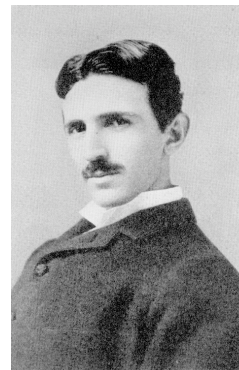
UNITED STATES PATENT OFFICE.

NIKOLA TESLA, OF NEW YORK, N. Y.

ART OF TRANSMITTING ELECTRICAL ENERGY THROUGH THE NATURAL MEDIUMS.

SPECIFICATION forming part of Letters Patent No. 787,412, dated April 18, 1905.

Application filed May 16, 1900. Renewed June 17, 1902. Serial No. 112,094.



Tesla applied for his patent in 1900; it was granted in 1905. Important excerpts follow:

Tesla does not know about the ionosphere or conduction in the atmosphere. He treats the Earth as a perfectly conducting sphere.

ical rigor. For the present it will be sufficient to state that the planet behaves like a perfectly smooth or polished conductor of inappreciable resistance with capacity and self induction uniformly distributed along the axis of symmetry of wave propagation and transmitting slow electrical oscillations without sensible distortion and attenuation.

22

30 First. The earth's diameter passing through the pole should be an odd multiple of the quarter wave length—that is, of the ratio between the velocity of light—and four times the frequency of the currents.

Tesla seems to be thinking of propagation through the Earth;
 $\square = (2n+1)c/8R$;
 $\approx 5.9 (2n+1) \text{ Hz}$

He is thinking of power transmission, not radiation into space, and so is keeping the frequency down, 6 Hz being the minimum.

magnetic waves is very small. To give an idea, I would say that the frequency should be
 40 smaller than twenty thousand per second, though shorter waves might be practicable. The lowest frequency would appear to be six per second, in which case there will be but one node, at or near the ground-plate, and, par-

Third. The most essential requirement is, however, that irrespective of frequency the wave or wave-train should continue for a certain interval of time, which I have estimated to be not less than one-twelfth or probably 0.08484 of a second and which is taken in passing to and returning from the region diametrically opposite the pole over the earth's
 55 surface with a mean velocity of about four hundred and seventy-one thousand two hundred and forty kilometers per second.

The stated speed is $v/c = \pi/2$. It makes the time taken over the surface from pole to pole equal to the time at speed c along the diameter. *Puzzling!*

23

FitzGerald presented a paper at the annual meeting of the British Association in September 1893.

Report of the British Association for the Advancement of Science, 63, 682 (1893)

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REPORT—1893.

6. *On the Period of Vibration of Electrical Disturbances upon the Earth.*
 By Professor G. F. FITZGERALD, Sc.D., M.A., F.R.S., F.T.C.D.

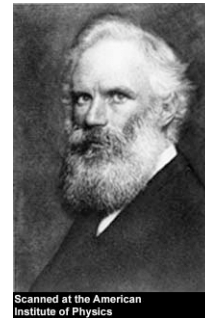
Professor J. J. Thomson and Mr. O. Heaviside have calculated the period of vibration on a sphere alone in space and found it about .059 second. The fact that the upper regions of the atmosphere conduct makes it possible that there is a period of vibration due to the vibrations similar to those on a sphere surrounded by a concentric spherical shell.

. The value of the time of vibration obtained by this very simple approximation is

$$T = \pi \sqrt{\frac{2K\mu a^2 b^4 \log a/b}{a^2 - b^2}}$$

Applying this to the case of the earth with a conducting layer at a height of 100 kilometres (much higher than is probable) it appears that a period of vibration of about one second would be possible. A variation in the height of the conducting layer produces only a small effect upon this if the height be small compared with the diameter of the earth.

$$T \approx \pi b/c \approx 1/15 \text{ Hz}^{-1} \text{ if } a \approx b$$



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FitzGerald had the proper model of the resonant cavity. He was off by $\sqrt{2}$ in the frequency.

24

In the report of the BA meeting in the September 28, 1893 issue of *Nature*, the reporter notes that “Prof. G. F. Fitzgerald gave an interesting communication on ‘The period of vibration of Disturbances of Electrification of the earth.’” He notes the following points made by FitzGerald:

1. “..... the hypothesis that the Earth is a conducting body surrounded by a non-conductor is not in accordance with the fact. Probably the upper regions of our atmosphere are fairly good conductors.”
2. “..... we may assume that during a thunderstorm the air becomes capable of transmitting small disturbances.”
3. “If we assume the height of the region of the aurora to be 60 miles or 100 kilometers, we get a period of oscillation of 0.1 second.”

In 1893 FitzGerald had the right model, got roughly the right answer for the lowest mode, and had the prescience to draw attention to thunderstorms, the dominant method of excitation of Schumann resonances.

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Example No. 4

Weizsäcker-Williams Method of Virtual Quanta

Carl Friedrich von Weizsäcker, 1934

Evan James Williams, 1934, 1935

Enrico Fermi, 1924

26

In 1924, before Quantum Mechanics, Enrico Fermi addressed the excitation of atoms in collisions with electrons and energy loss of charged particles in a novel way:



Zeitschrift für Physik, **29**, 315-327 (1924)

Über die Theorie des Stoßes zwischen Atomen und elektrisch geladenen Teilchen.

Von **E. Fermi** in Rom.

(Eingegangen am 20. Oktober 1924.)

Das elektrische Feld eines geladenen Teilchens, welches an einem Atom vorbeifliegt, wird harmonisch zerlegt, und mit dem elektrischen Feld von Licht mit einer passenden Frequenzverteilung verglichen. Es wird angenommen, daß die Wahrscheinlichkeit, daß das Atom vom vorbeifliegenden Teilchen angeregt oder ionisiert wird, gleich ist der Wahrscheinlichkeit für die Anregung oder Ionisation durch die äquivalente Strahlung. Diese Annahme wird angewendet auf die Anregung durch Elektronenstoß und auf die Ionisierung und Reichweite der α -Strahlen.

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A rough literal translation is

“The electric field of a charged particle that passes by an atom, when decomposed into harmonics, is equivalent to the electric field of light with an appropriate frequency distribution. It will be assumed that the probability that an atom will be excited or ionized by the passing particle is equal to the probability for excitation or ionization through the equivalent radiation. This hypothesis will be applied to the excitation through electron collisions and to the ionizing power and range of α -particles.”

That first sentence describes a key ingredient of the Weizsäcker-Williams method of virtual quanta.

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Nine years later, E. J. Williams discussed the limitations of Fermi's work in the light of quantum mechanics. A year later, in Part III of a Letter to Phys. Rev. (**45**, 729-730 (1934)), Williams said

.....
 Practically the same considerations apply to the formula of Heitler and Sauter⁶ for the energy lost by an electron in radiative collisions with an atomic nucleus. C. F. v. Weizsäcker, and the writer, in calculations shortly to appear elsewhere, show that this formula may readily be derived by considering, in a system S' where the electron is initially at rest, the scattering by the electron of the harmonic components in the Fourier spectrum of the perturbing force due to the nucleus (which, in S' , is the moving particle). The calculations show that practically all the radiative energy loss comes from the scattering of those components with frequencies $\sim mc^2/h$, and also that Heitler and Sauter's formula is largely free from the condition $Ze^2/hc \ll 1$, which generally has to be satisfied in order that Born's approximation (used by H and S) may be valid.



E. J. Williams

Virtual quanta of nucleus moving by electron in its rest frame are Compton-scattered to give bremsstrahlung. Hard photons in the lab come from $h\nu \sim mc^2$ in rest frame S'

29



Carl F. von Weizsäcker

Von Weizsäcker, in *Z. f. Physik*, **88**, 612- 625 (1934), and in more generality by Williams, in "Correlation of certain collision problems with radiation theory," *Kgl. Danske Videnskab. Selskab Mat.-fys. Medd.*, **XII**, No.4, (1935),

exploited special relativity to show that in very high energy radiative processes the dominant energies are- always of order of the light particle's rest energy when seen in the appropriate rest frame. The possible failure of quantum electrodynamics at extreme energies posited by Oppenheimer and others does not occur (muons!).

Fermi started it; Williams obviously knew of Fermi's virtual photons; he and Weizsäcker chose the right rest frames for relativistic processes. The "Weizsäcker-Williams method" continues to have wide and frequent applicability.

30

Example No. 5

BMT Equation for Spin Precession in E-M Fields

Valentine Bargmann, Louis Michel,
And Valentine L. Telegdi, 1959

Llewellyn Hilleth Thomas, 1927

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THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.
[SEVENTH SERIES.]
JANUARY 1927.

I. *The Kinematics of an Electron with an Axis.*
By L. H. THOMAS, B.A., Trinity College, Cambridge *.

A detailed 22-page paper to elaborate
on his short letter to *Nature* in 1926.

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PRECESSION OF THE POLARIZATION OF PARTICLES MOVING IN A HOMOGENEOUS
ELECTROMAGNETIC FIELD*

V. Bargmann

Princeton University, Princeton, New Jersey

Louis Michel

Ecole Polytechnique, Paris, France

and

V. L. Telegdi

University of Chicago, Chicago, Illinois

(Received April 27, 1959)



Bargmann



Michel



Telegdi

Scanned at the American
Institute of Physics**Equation of motion for particle of charge e**

Thomas, 1927

$$\frac{d}{ds} \left(m \frac{dx^\mu}{ds} \right) = - \frac{e}{c} F^\mu{}_\nu \frac{dx^\nu}{ds} \cdot \cdot \cdot \cdot \cdot \quad (1.73)$$

BMT, 1959

electromagnetic field specified by $F = -(\vec{E}, \vec{H})$

$$du/d\tau = (e/m)F \cdot u. \quad (5)$$

For the spin motion, Thomas used both a spin 4-vector w and an antisymmetric second-rank tensor $w^{\mu\nu}$.

Equation for spin motion in external fields

BMT, 1959

With (5), one has for homogeneous fields

$$ds/d\tau = (e/m)[(g/2)F \cdot s + (g/2 - 1)(s \cdot F \cdot u)u]. \quad (7)$$

Thomas, 1927

In this case, to the same approximation,

Here $g = 2$

$$\frac{dw^\mu}{ds} = \frac{e}{mc} F^\mu{}_\nu w^\nu, \quad \dots \dots \dots (4.123)$$

and

$$\frac{dw_{\mu\nu}}{ds} = \frac{e}{mc} \{ F^\sigma{}_\nu w_{\sigma\mu} - F^\sigma{}_\mu w_{\sigma\nu} \}. \quad \dots \dots (4.124)$$

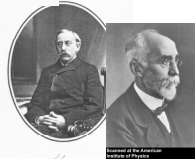
($g \neq 2$)

BUT!!

The more complicated forms when $\lambda \neq e/mc$ involving \mathbf{v} explicitly on the right-hand side can be found easily if required.

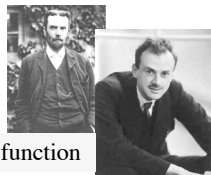
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Lorenz

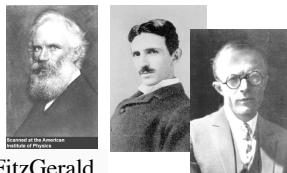


Lorentz condition

Heaviside



Dirac delta function



FitzGerald

Schumann resonances

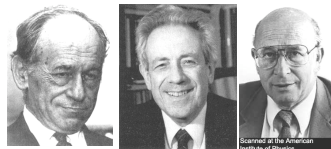
The First
versus
The Famous



Fermi

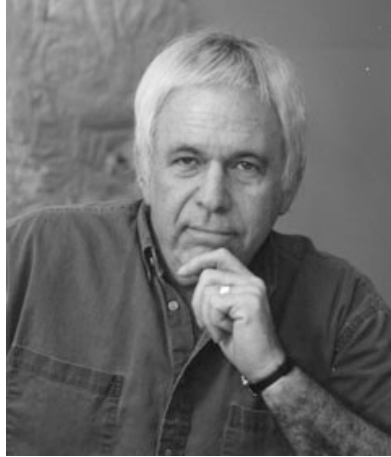
Weizsäcker-Williams method

Thomas

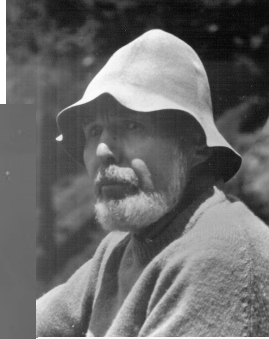


BMT equation for spin motion

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January 19, Kane's birthday, 1937



b. 01.19. <1937

Happy
Birthday
Gordy !